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31 Dec 1963, DoDD 5200.10; BUSHIPS ltr, 1 Apr 1968	

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PROGRESS REPORT

AND

INTERIM REPORT

AD

CAPEHART - FARNSWORTH CORPORATION

FORT WAYNE, INDIANA

CAPEHART-FARNSWORTH CORPORATION

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December 13, 1951 Progress Report

Contract NObsr-42080

Status of Work December 1, 1951

Summary:

(1) Progress Report includes Interim Report, "Thermal Constants of Thin Films".

(2) Exploration of possible optical systems for test of sealed off suggests use of BuOrd system in spite of inherent distortion when image is focused on plane surface.

(3) Delivery of low temperature source IR system scheduled for early in January.

Optical Systems:

As indicated in the October report a special aspheric corrector plate must be employed in a reflective optical system if an abberative free image is to be obtained on a plane surface. (The glass film target cannot be made curved). Mr. Hunt, of BuShips, has explored the possibility of locating just such a system in the various bureaus but the indications to date are that said system is not existant. As a result it has been verbally agreed that BuShips will procure for us one BuOrd system with silver chloride corrector plate for 4" radius spherical image surface. Only a small portion of the field will be used in order to minimize the abberation. This still will permit final check of sensitivity and resolution of a sealed off glass film orthicon. The results should indicate whether design and building of a new optical system is warranted.

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I.R. Test System:

The vacuum low temperature source and optical system will give an absolute figure of sensitivity. This piece of equipment is scheduled for delivery early in January but will require several weeks for installation and alignment before this data can be taken.

Approved:

S. F. Essig, Project Engineer

CAPEHART-FARNSWORTH CORPORATION

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THERMAL CONSTANTS OF THIN FILMS

INTRODUCTION

This paper is a summary of the important theoretical and experimental results obtained in the development of thin film heat detectors. Our main purpose will be to outline clearly what has been done in order to see along what lines further experimental research can be directed. The methods of determining the quantities which enter into the theoretical discussion are illustrated and data are presented showing results obtained on Corning 008 glass films.

Previous experimental results have yielded a value of thermal conductivity from 10 to 20 times the bulk value of the thermal conductivity of 008 glass. This discrepancy has been rechecked by a more thorough investigation leading to values of thermal conductivity as low as twice the bulk value. In view of the fact that the results depend upon the operating conditions of the orthicon and the theory applies only to small signals, the results are considered as showing a fair agreement between theory and experiment.

A previous report[1)] described the steady state problem of the distribution of temperature in a thin film for a given radiation input. The report[1)] states the fundamental assumptions made in deriving a heat conduction equation and also discusses the solutions to the equation for various input patterns of radiation. The equation and the solution for a single

1) The Distribution of Temperature in Thin Films: Research Memo 189.

bar or strip pattern is used as a starting point for this discussion. The bar pattern leads to a relatively simple one dimensional equation whose solution can be written in closed form. This is a great advantage for computational purposes and explains the emphasis on this particular input pattern.

The equation is:

$$\nabla^2 T - \beta^2 T + \beta^2 \mu f(x,y) = 0, \quad f(x,y) = \text{radiation input function} \tag{1}$$

where

$$\mu = \frac{I}{8 \epsilon \sigma T_0^3} \tag{2}$$

$$\beta^2 = \frac{8 \sigma \epsilon T_0^3}{\lambda \delta} \tag{3}$$

ϵ = emissivity of surface

σ = Stefans constant

T_0 = ambient temperature

T = temperature rise of surface above ambient (considered a small quantity)

I = intensity of incoming energy

λ = heat conductivity of film

δ = thickness of film

The solution to equation (1) for a single bar pattern of width 2w, infinite length and uniform intensity, focussed on an infinite plane screen is given by:

$$(4) \quad T = \mu\left[1 - \frac{e^{-\beta w}}{2}\left(e^{\beta x} + e^{-\beta x}\right)\right] \text{ for } x < w$$

$$(5) \quad T = \mu\left[\frac{e^{\beta w} - e^{-\beta w}}{2}\right] e^{-\beta x}$$

where $2w$ = the width of the bar pattern

x = the coordinate measured perpendicular to the length of the bar of illumination

It can be shown that the edge effect due to the finite size of the target film is small, much less than 1 percent, providing the width of the bar is kept to say one-quarter the diameter of the screen.

Equation (5) gives the distribution of temperature outside of the radiation pattern and this rise in temperature is due to the sidewise heat conduction of heat from the illuminated strip. The sharpness of a thermal image will depend upon how fast the "wings" of the temperature distribution die out to small values with increasing x. Equation (5) shows that β will determine how fast the temperature rise of the film reaches a negligible value outside of the radiation pattern. For this reason β is called the shaping factor and for a sharp thermal image β should be as large as possible. Considering the ambient temperature and the conductivity as fixed, equation (3) indicates a thickness as small as possible and an emissivity as large as possible for a maximum β.

A check on the theory will be if an experimentally determined β, ε, and δ leads, through equation (5), to a value of λ in agreement with the known values of bulk conductivity for the materials composing the film. The value of δ is obtained by a standard interferometric method and the method will not be considered here. Methods accounting for the effects of the chromium backing will be described and experimental evidence showing the effects of the backing presented.

TRANSIENT CASE

An investigation of the transient case is particularly useful since one can expect to get a measure of emissivity from the decay time of a flooded film. The reason is that a flooded film will lose heat primarily by radiation and the rate at which the film loses heat is dependent on the emissivity.

There are two cases to be considered.

CASE 1: The growth of the heat pattern upon application of the radiation pattern. We again consider a strip pattern of width 2w. The equation and boundary conditions for this case are:

$$\frac{\partial^2 T}{\partial x^2} - \beta^2 T + \beta^2 \rho \lambda f(x) = \frac{c\rho}{\lambda}\frac{\partial T}{\partial t}$$

C = specific heat

ρ = density

t = time

The initial condition is:

$$T = 0 \text{ when } t = 0$$

The boundary condition is:

$$T \to 0 \text{ when } \pm x \to \infty$$

where

$$f_{(x)} = \begin{cases} 1 & \text{when } -w < x < w \\ 0 & \text{when } -w > x > w \end{cases}$$

CASE 2: The cooling of a film when the bar of radiation is removed. The equation and boundary and initial conditions are:

$$\frac{\partial^2 T}{\partial x^2} - \beta^2 T = \frac{c\rho}{\lambda} \frac{\partial T}{\partial t}$$

The initial condition is:

$$T_0 = \mu\left[1 - \frac{e^{-\beta w}}{2}\left(e^{\beta x} + e^{-\beta x}\right)\right] \text{ for } x < w$$

$$T_0 = \mu\left[\frac{e^{\beta w} - e^{-\beta w}}{2}\right] e^{-\beta x} \text{ for } x > w$$

and

$$T \to 0 \text{ when } \pm x \to \infty$$

Because Case 2 does not contain an inhomogeneous term in the differential equation, it can be solved in a straight forward manner. For the sake of completeness the solution will be recorded here, but the details can be skipped without losing continuity. One has the equation

$$\frac{\partial^2 T}{\partial x^2} - \beta^2 T = \frac{c\rho}{\lambda} \frac{\partial T}{\partial t}$$

Let $T = \theta e^{-\beta^2 \frac{\lambda}{c\rho} t}$ and substituting we have

$$\frac{\partial^2\Theta}{\partial x^2} = \frac{c\rho}{\lambda}\frac{\partial\Theta}{\partial t} = \frac{1}{K}\frac{\partial\Theta}{\partial t}$$

or

(6) $$K\frac{\partial^2\Theta}{\partial x^2} = \frac{\partial\Theta}{\partial t}$$

Where the initial distribution of temperature is given by the temperature distribution for a bar pattern, i.e., when t = 0

$$\Theta_0 = \mu\left[1 - \frac{e^{-\beta\omega}}{2}\left(e^{\beta x}+e^{-\beta x}\right)\right]$$

(7)

$$\Theta_0 = \mu\left[\frac{e^{\beta\omega}-e^{-\beta\omega}}{2}\right]e^{-\beta x}$$

By substitution it can be shown that the solution to equation (6) is given by

(8) $$\Theta = \frac{1}{\sqrt{\pi K t}}\cdot\int_0^{+\infty}\Theta_0(x')\,e^{-\frac{(x-x')^2}{4Kt}}dx'$$

Substituting equation (7) in equation (8) we have

$$\Theta(x,t) = \frac{\mu}{\sqrt{\pi K t}}\int_0^{\omega}\left[1-\frac{e^{-\beta\omega}}{2}\left(e^{\beta x'}+e^{-\beta x'}\right)\right]e^{-\frac{(x-x')^2}{4Kt}}dx'$$

(9)

$$+\frac{\mu}{\sqrt{\pi K t}}\;\frac{e^{\beta\omega}-e^{-\beta\omega}}{2}\int_{\omega}^{\infty}e^{-\beta x'}e^{-\frac{(x-x')^2}{4Kt}}dx'$$

We concern ourselves now with the behaviour of the temperature with time at the point x = 0. Thus equation (9) becomes

$$\frac{\Theta(0,t)\sqrt{\pi K t}}{\mu} = \int_0^{\omega} e^{-x'^2/4Kt}dx' - e^{-\beta\omega}\int_0^{\omega} e^{\beta x'}e^{-\frac{x'^2}{4Kt}}dx'$$

$$-\frac{e^{-\beta\omega}}{2}\int_0^{\omega} e^{-\frac{x'^2}{4Kt}}e^{-\beta x'}dx' + \frac{e^{\beta\omega}-e^{-\beta\omega}}{2}\int_{\omega}^{\infty} e^{-\beta x'}e^{-\frac{x'^2}{4Kt}}dx'$$

By changing the variables of integration each of the four integrals can be transformed as follows:

$$\int_0^{\omega} e^{-\frac{x'^2}{4Kt}}dx' = 2\sqrt{Kt}\int_0^{\frac{\omega}{2\sqrt{Kt}}} e^{-\xi^2}d\xi$$

$$\int_0^{\omega} e^{\beta x'}e^{-\frac{x'^2}{4Kt}}dx' = e^{\beta^2 Kt}\int_0^{\omega} e^{-\beta^2 Kt + 2\frac{\beta\sqrt{Kt}}{2\sqrt{Kt}}x' - \frac{x'^2}{4Kt}}dx'$$

$$= 2\sqrt{Kt}\,e^{\beta^2 Kt}\int_{-\beta\sqrt{Kt}}^{\frac{\omega}{2\sqrt{Kt}}-\beta\sqrt{Kt}} e^{-\xi^2}d\xi$$

$$\int_0^{\omega} e^{-(\beta x' + \frac{x'^2}{4Kt})}dx' = 2\sqrt{Kt}\,e^{\beta^2 Kt}\int_{\beta\sqrt{Kt}}^{\frac{\omega}{2\sqrt{Kt}}+\beta\sqrt{Kt}} e^{-\xi^2}d\xi$$

$$\int_{\omega}^{\infty} e^{-(\beta x' + \frac{x'^2}{4Kt})}dx' = 2\sqrt{Kt}\,e^{\beta^2 Kt}\int_{\frac{\omega}{2\sqrt{Kt}}+\beta\sqrt{Kt}}^{\infty} e^{-\xi^2}d\xi$$

Thus the expression for Θ becomes

$$\Theta(0,t)\frac{\sqrt{\pi}}{\mu} = 2\int_0^{\frac{\omega}{2\sqrt{Kt}}} e^{-\xi^2}d\xi - e^{-\beta\omega}e^{\beta^2 Kt}\int_{-\beta\sqrt{Kt}}^{\frac{\omega}{2\sqrt{Kt}}-\beta\sqrt{Kt}} e^{-\xi^2}d\xi$$

$$- e^{-\beta\omega}e^{\beta^2 Kt}\int_{\beta\sqrt{Kt}}^{\frac{\omega}{2\sqrt{Kt}}+\beta\sqrt{Kt}} e^{-\xi^2}d\xi + (e^{\beta\omega}-e^{-\beta\omega})e^{\beta^2 Kt}\int_{\frac{\omega}{2\sqrt{Kt}}+\beta\sqrt{Kt}}^{\infty} e^{-\xi^2}d\xi$$

Since $f = \theta e^{-\beta^2 k t}$ we have

$$\frac{T(0,t)\sqrt{\pi}}{\mu} = 2e^{-\beta^2 kt}\int_0^{\frac{\omega}{2\sqrt{kt}}} e^{-\xi^2}d\xi - e^{\beta\omega}\int_{-\beta\sqrt{kt}}^{\frac{\omega}{2\sqrt{kt}}-\beta\sqrt{kt}} e^{-\xi^2}d\xi$$

$$- e^{-\beta\omega}\int_{\beta\sqrt{kt}}^{\frac{\omega}{2\sqrt{kt}}+\beta\sqrt{kt}} e^{-\xi^2}d\xi + \left(e^{\beta\omega}-e^{-\beta\omega}\right)\int_{\frac{\omega}{2\sqrt{kt}}+\beta\sqrt{kt}}^{\infty} e^{-\xi^2}d\xi$$

If we let $w \to \infty$, i.e., flood the film with radiation, we get a very useful result since the last three integrals approach zero while

$$\lim_{\omega\to\infty}\int_0^{\frac{\omega}{2\sqrt{kt}}} e^{-\xi^2}d\xi = \frac{\sqrt{\pi}}{2}$$

Thus

$$\frac{T(0,t)}{\mu} = e^{-\beta^2 kt} = e^{-\gamma^2 t} \tag{10}$$

for a flooded film where

$$\beta^2 k = \gamma^2$$

If $t_{1/2}$ is the time necessary for the temperature of the flooded film to fall to one-half maximum, we have

$$\gamma^2 = \frac{\log_e 2}{t_{\frac{1}{2}}} \doteq \frac{0.69}{t_{\frac{1}{2}}} \tag{11}$$

Since $\beta^2 = \dfrac{8\epsilon\sigma T_0^3}{\lambda\delta}$ and $K = \dfrac{\lambda}{c\rho}$

then $\gamma^2 = \dfrac{8\epsilon\sigma T_0^3}{\delta c\rho}$

or

(12) $\epsilon = \dfrac{\gamma^2\delta c\rho}{8\sigma T_0^3} = \dfrac{\log_e 2\,\delta c\rho}{t_{\frac{1}{2}}\,8\sigma T_0^3}$

Equation (12) enables us to make a measurement of the emissivity of the film.

This same result for the emissivity has been derived in a British report. (Addendum to C.V.D., Quarterly Report No. 17) from different considerations. Suppose we consider no sidewise conduction then the rate at which an area loses heat is given by

$$8\epsilon\sigma T_0^3 T$$

The rate of heating per unit area of film is equal to

$$\delta c\rho\frac{dT}{dt}$$

If K = rate of absorption of radiation then the balance of energy is

$$\frac{dT}{dt} + \frac{8\epsilon\sigma T_0^3}{\delta c\rho}T = \frac{K}{\delta c\rho}$$

The solution to this equation if T = 0 when t = 0 is

$$T = \frac{K}{8\epsilon\sigma T_0^{\,3}}\left(1 - \exp - \left\{\frac{8\epsilon\sigma T_0^{\,3}}{\delta c\rho}\, t\right\}\right)$$

This equation applies to a flooded film because that situation should approximate no sidewise heat flow. The time constant is just the same as the value previously obtained.

Equation (12) enables us to make a measurement of emissivity of the film. This measurement does not depend upon λ or β and requires only the measurement of $t_{1/2}$. The values of $C\rho$ and σ can be obtained from tables of physical constants.

DETERMINATION OF β

<u>Experimental Measurement of</u>

Consider the steady state solutions to the heat conduction equation (1) for a bar pattern.

$$T = \mu\left[1 - \frac{e^{-\beta w}}{2}\left(e^{\beta x} + e^{-\beta x}\right)\right] \text{ for } x < w \qquad (4)$$

$$T = \mu\left[\frac{e^{\beta w} - e^{-\beta w}}{2}\right] e^{-\beta x} \text{ for } x > w \qquad (5)$$

Now consider only the point x = 0, then from equation (4) we have

(13) $$T_{max} = \mu(1 - e^{-\beta w})$$

For a flooded film, i.e., $w \to \infty$, we have

(14) $$T_{fl} = \mu$$

Taking the ratio of the above expressions we have

(15) $$T_{max}/T_{fl} = 1 - e^{-\beta w}$$

Furthermore we assume the current i, resistance R of film, and temperature T are proportional

$$i \propto R \propto T$$

Therefore

(16) $$i_{max}/i_{fl} = 1 - e^{-\beta w}$$

From the orthicon and line scan apparatus described in previous reports we obtain the values for the ratio of currents i_{max}/i_{fl}. Figure 1 is a typical experimental measurement. The interpretation of the current traces on the oscilloscope depends on the assumptions that changes in temperature are small and that the orthicon is operating under the correct operating conditions. The bar radiation pattern is scanned perpendicular to its length giving the rounded peak. Then the lower trace is taken with no incident radiation. The current corresponds to ambient temperature. Finally the film is flooded with radiation, having the same intensity as the bar pattern. The current corresponding to the flooded film is represented by the upper trace. The required ratio of i_{max}/i_{fl} is taken from the oscillograph traces. βw can be obtained directly from equation (16) and since w is known β can be determined.

Experimental Measurement of ε

In the previous section it is shown in great detail how the emissivity of the film is related to the decay time of the temperature of a flooded film.

Figure 2 shows a typical oscillograph trace from which ε can be obtained. By calibrating the sweep frequency of the orthicon electron beam, the time interval between sweeps is known. Thus the number of traces between maximum current and one-half maximum gives $t_{1/2}$ and ε is obtained by substitution in equation (12).

Measurement of λ_{av}

From the results previously obtained we can derive an average heat conductivity for a composite film.

We have from equations (3) and (12)

$$\beta^2 = \frac{8\epsilon\sigma T_0^3}{\lambda\delta} \qquad (13)$$

$$(17) \qquad \epsilon = \frac{\log_e 2\,\delta c\rho}{8\sigma T_0^3 t_{\frac{1}{2}}}$$

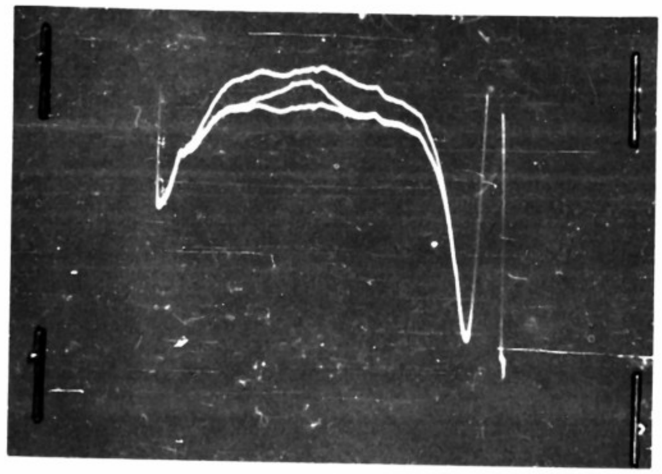


Figure 1

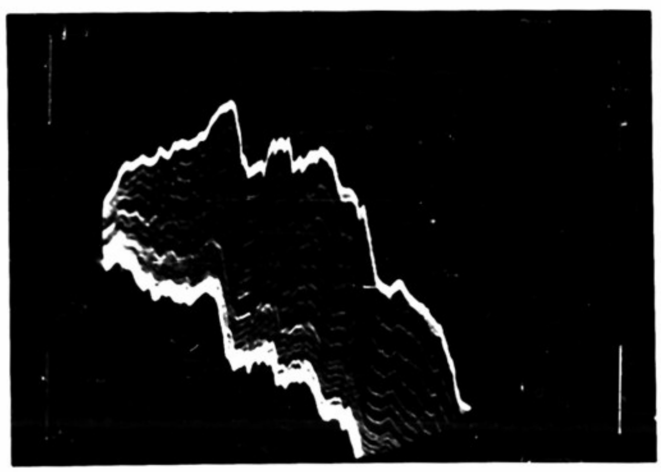


Figure 2

The product of the two expressions yield

$$\beta^2 = \frac{\log_e 2\, c\rho}{t_{\frac{1}{2}}\, \lambda}$$

or

$$\lambda_{av} = \frac{\log_e 2\, c\rho}{t_{\frac{1}{2}}\, \beta^2} \tag{18}$$

Since the quantities $c\rho$ for glass, metal, and lampblack do not differ greatly, and the glass forms the major part of the mass of the film, the value of $c\rho$ for glass may be used with small error. The quantities β and $t_{1/2}$ are obtained directly from experiments. Ideally the value of λ_{av} should be that of glass. However, the effects of the backing materials increase the values of λ_{av} above that of bulk glass. Note that equation (18) does not depend on a measurement of thickness.

SUMMARY OF RESULTS

Now the basic formulae are stated and they must be interpreted in terms of the physical situation. The films are not homogeneous, but consist in general of a glass film with a conductive backing, usually evaporated chromium, and a radiation absorbing coating. The question now arises, what is the interpretation of the quantities ε, δ, and λ which enter into the formulae and how are they related to the experimentally measured values of β, the thickness of the glass film, the blackening layer, and the decay time of a flooded film. As an aid in discussion the two equations used for computation are:

$$(19)\quad \lambda = \frac{8\epsilon\sigma T_0^{3}}{\beta^2\delta}$$ rearranging equation (3)

$$\epsilon = \frac{\log_e 2\,\delta c\rho}{8\sigma T_0^{3}\, t_{\frac{1}{2}}} \quad (17)$$

and

$$\lambda_{av} = \frac{\log_e 2\, c\rho}{t_{\frac{1}{2}}\,\beta} \quad (18)$$

The values of the constants appearing in (17), (18), and (19) used for numerical work are:

$$\ln 2 = 0.69$$

$$c\rho = 0.48\ \frac{\text{cal}}{\text{cm}^3\ {}^\circ\text{C}} = 2.01\ \frac{\text{watts}-\text{sec}}{\text{cm}^3\ {}^\circ\text{C}}$$

$$\sigma = 5.7 \times 10^{-12}\ \frac{\text{watts}}{\text{cm}^2\ {}^\circ\text{C}^4}$$

$$T_o = 300^\circ\text{K}$$

The heat conductivity of chromium is

$$\lambda_{\text{chromium}} = 0.69\ \frac{\text{watts}}{\text{cm}\ {}^\circ\text{C}} = 0.165\ \frac{\text{cal}}{\text{sec cm}\ {}^\circ\text{C}}$$

The heat conductivity of the bulk glass is taken as

$$\lambda_{\text{glass}} = 0.002\ \frac{\text{cal}}{\text{sec cm}\ {}^\circ\text{C}}$$

Note that equation (18) is derived from equation (19) by substitution of the value of ε given by equation (17). Equation (18) is merely a convenient forumula, but does not contain anything new. The quantity to be calculated is the conductivity of glass using experimental values of $t_{1/2}$, β, and δ. We neglect the effects of the blackening layer and consider only the composite film, glass plus chromium. Further the chrome layer is less than 2 percent of the total thickness of the film, thus within the limits of experimental accuracy the thickness of the glass film is taken to be the thickness of the composite film. We now proceed to calculate the conductivity of glass in two ways. The methods will be shown to be the same.

METHOD 1: The basis for this calculation is the assumption that the heat conductivity of the composite film is given by the weighted average

$$\lambda_{av} = \lambda_{glass} \frac{\delta_{glass}}{\delta} + \lambda_{metal} \frac{\delta_{metal}}{\delta} \tag{20}$$

Since $\delta_{glass} >> \delta_{metal}$ we can write

$$\lambda_{av} = \lambda_{glass} + \lambda_{metal} \frac{\delta_{metal}}{\delta} \tag{21}$$

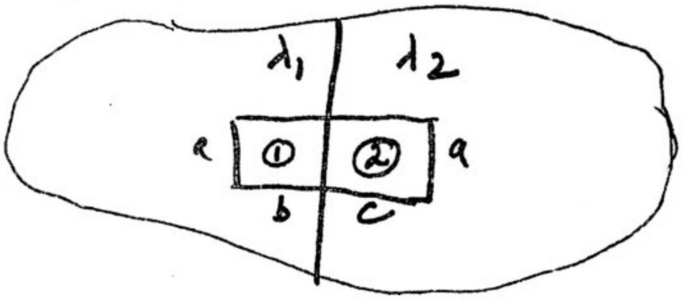
Using this equation and equation (18) the values of λ_{glass} can be computed.

METHOD 2: The basis for this calculation is the assumption that for purposes of heat conduction we can replace the composite film by a glass film having an effective thickness defined in a reasonable way. The effective thickness of the composite film is defined as the sum of the real thickness of the glass film plus a fictitious thickness of glass, which would conduct the same amount of heat sidewise as the given layer of chromium. The effective thickness is given by

$$\delta_{eff} = \delta + \frac{\lambda_{metal}}{\lambda_{glass}} \delta_{metal} \tag{22}$$

This effective thickness and equation (19) are used to calculate λ.

We now show that the two assumptions (20) and (22) are equivalent. Consider the boundary between two semi-infinite media 1 and 2 having heat conductivities λ_1 and λ_2 respectively.



Assume a unit temperature gradient normal to the paper. Thus the rate of heat flow across rectangles ab and ac are

$$\frac{dQ_1}{dt} = -\lambda_1 ab \quad \text{and} \quad \frac{dQ_2}{dt} = -\lambda_2 ac \quad \text{respectively.}$$

The sum is

$$\frac{dQ_1}{dt} + \frac{dQ_2}{dt} = -(\lambda_1 ab + \lambda_2 ac).$$

Now using equation (20) we define an average heat conductivity λ_{av} by

$$\lambda_{av} = \lambda_1 \frac{b}{b+c} + \lambda_2 \frac{c}{b+c}$$

Using this average heat conductivity calculate the rate of heat flow.

$$\frac{dQ}{dt} = -(\lambda_1 \frac{b}{b+c} + \lambda_2 \frac{c}{b+c})\ a(b+c)$$

$$= -(\lambda_1 ab + \lambda_2 ac)$$

Using equation (22) we define a fictitious size for rectangle 2. We have for the effective value of C

$$\frac{\lambda_2}{\lambda_1} C.$$

Using the value λ_1 for the heat conductivity of the fictitious rectangle of area

$$a(b + \frac{\lambda_2}{\lambda_1} C)$$

We have for the rate of heat flow

$$\frac{dQ}{dt} = -\lambda_1 a(b + \frac{\lambda_2}{\lambda_1} C) = -(\lambda_1 ab + \lambda_2 aC).$$

Thus the methods are equivalent.

The problem now lies in the interpretation of the quantities appearing in equations (18) and (19). Using equation (19) and substituting for δ, the effective thickness of the film, we have

$$\lambda_{glass} = \frac{\eta\,\varepsilon}{\beta^2\left(\delta + \frac{\lambda_{metal}}{\lambda_{glass}}\,\delta_{metal}\right)}$$

$$= \frac{\eta\,\varepsilon}{\beta^2\delta\left(1 + \frac{\lambda_{metal}}{\lambda_{glass}}\,\frac{\delta_{metal}}{\delta}\right)}$$ where $\eta = 8\sigma T_0^3$

Thus

$$\lambda_{glass}\left(1 + \frac{\lambda_{metal}}{\lambda}\,\frac{\delta_{metal}}{\delta}\right) = \eta\,\frac{\varepsilon}{\beta^2\delta}$$

$$\therefore \quad \lambda_{glass} = \eta\,\frac{\varepsilon}{\beta^2\delta} - \lambda_{metal}\,\frac{\delta_{metal}}{\delta}$$

Remembering $\delta_{glass} >> \delta_{metal}$

Thus $\delta \doteq \delta_{glass}$

Or we have from equation (18), substituting for the average thermal conductivity,

$$\lambda_{glass} + \lambda_{metal}\,\frac{\delta_{metal}}{\delta} = \eta\,\frac{\varepsilon}{\beta^2\delta}$$

$$\lambda_{glass} = \eta\,\frac{\varepsilon}{\beta^2\delta} - \lambda_{metal}\,\frac{\delta_{metal}}{\delta}$$

Now recalling the derivation of equation (18) we see that we can write

$$(23) \qquad \lambda_{av} = \frac{\ln 2 : \delta \cdot c\rho}{\beta^2 \delta'' t_{1/2}} \equiv \lambda_{glass} + \lambda_{metal} \frac{\delta_{metal}}{\delta}$$

Ostensibly $\delta' = \delta''$ and we get equation (18). Rewriting equation (23), the value for λ_{glass} is

$$(24) \qquad \boxed{\lambda_{glass} = \frac{\ln 2 \cdot C}{\beta^2 t_{1/2}} - \lambda_{metal} \frac{\delta_{metal}}{\delta}}$$

Equation (24) is the final result for the heat conductivity of glass and is used for the numerical calculations. Table I gives the values for the conductivity of glass as determined from equation (24). The values of λ glass indicate that the correction factor to the average heat conductivity, i.e., $\lambda_{metal} \cdot \frac{\delta_{metal}}{\delta}$ is not as large as it should be. This trend is shown in the data since the thinest chromium backings give the best agreement. However, considering the difficult experimental technique, the agreement with an expected value of 0.002 $\frac{cal}{sec\ {}^{o}C\ cm}$ is considered fair.

One more loose end to be considered is the effective value of $c\rho$ of a composite film. The previous calculations assume the value of $c\rho$ for glass is correct. The assumption is shown to be relatively good for if one again considers a weighted average one obtains for the composite film.

$$c\rho/_{av} = c\rho/_{glass} \frac{\text{mass of glass per unit area}}{\text{total mass}} + c\rho/_{metal} \frac{\text{mass of metal per unit area}}{\text{total mass}}$$

$$c\rho /_{av} = 0.484$$

which differs from the $c\rho$ of glass by less than 1 percent.

TABLE I

Film No.	δ (microns)	β (cm^{-1})	$t_{1/2}$ (sec)	Equivalent Thickness of Chrome Backing (microns)	ε	λ_{glass} $\frac{cal}{sec\ cm\ ^\circ C}$
16	4.6	4.0	2.0	1.5	0.26	0.009
136	2	4.5	0.92	1.5	0.25	0.017
235	5	5.1	1.65	0.15	0.34	0.0077
230	5	6.6	1.65	0.08	0.34	0.0045

TABLE II

Film No.	Method of Blackening	Emissivity
16	Camphor smoke	0.26
136	No blackening	0.25
235	Sprayed lampblack	0.34
106	Sprayed aquadag	0.30
248	Sprayed with medium resistance paint	0.23

EMISSIVITY OF THE FILMS

It is now clear that the limiting value of the heat conductivity is being approached. Thus we are left with two parameters to vary, i.e., ϵ and δ. A value of one micron has been reached for δ and any further decrease in thickness will be difficult to realize. The parameter that needs to be controlled is the emissivity. A series of experiments were carried out to determine the effects of various blackening materials on the emissivity of the films. Table II shows the results.

The results of Table II are interpreted as meaning the blackening of the films has no effect upon the rate at which a film will lose heat. Two inferences regarding the loss of heat from the film can be made. First, the glass film radiates to the backing and the backing then radiates to the surroundings. Second, the heat is carried by conduction to the backing and then radiated. Note that the second hypothesis requires that the conduction time be adjusted so as to make the emissivity appear to be the same as a film which is not blackened.

One more effect of interest is that the emissivity is a function of the thickness of the film. This follows directly from the fact that emissivity and absorptivity are equal. Thus a thicker film absorbs more radiation and thus emits more radiation. A measure of the transmission of clear glass films was obtained using a source kept at 160° C. and a thermopile. The results of this measurement are shown in Figure 3.

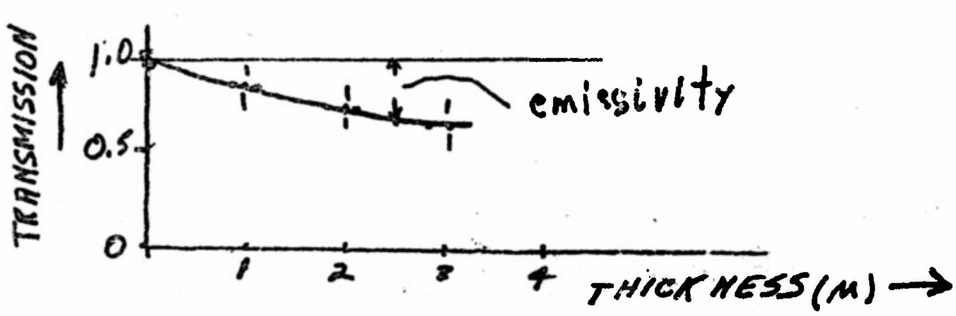


Figure 3

If we assume that the index of refraction of glass for infra-red is about the same as for visible light then 4% of the incident radiation is reflected. Extrapolating the experimental curve shows that the absorption quickly goes to zero for thicknesses less than one micron. This provides an additional reason for thinking that there is not a good reason for going to films thinner than one micron.

The values of emissivities obtained from the transmission experiment are consistent with the values found from the decay time.

It should be noted that while the blackening does not increase the rate at which the film loses heat, it does raise the temperature of the film in a radiation field above that of the unblackened film. Blackening is still necessary for a sensitive instrument.

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12-13-51